Integration by completing square method

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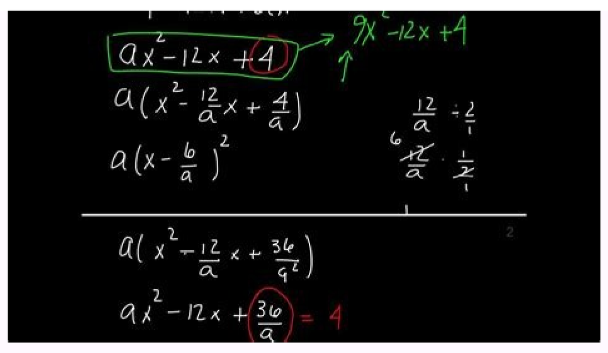

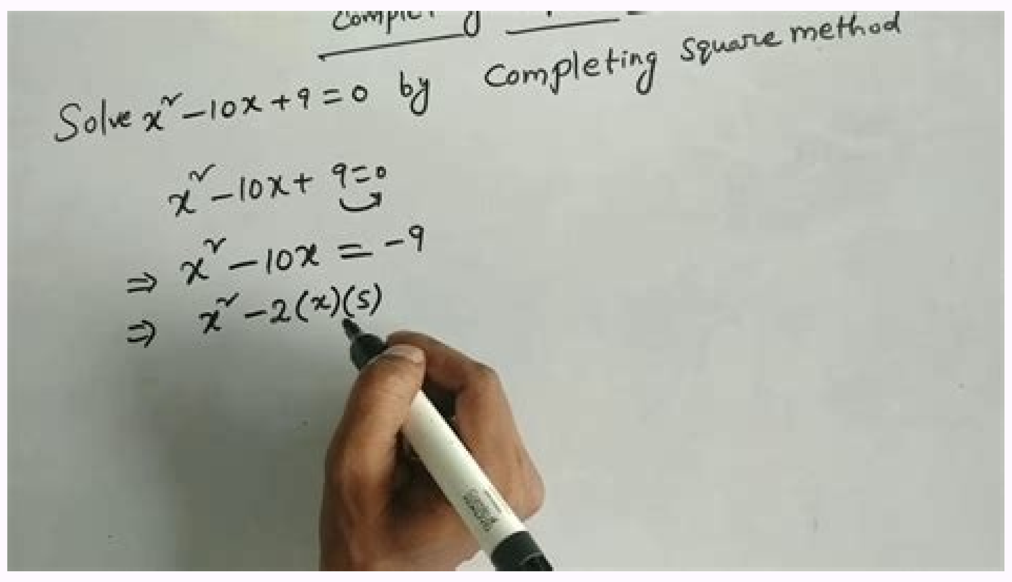

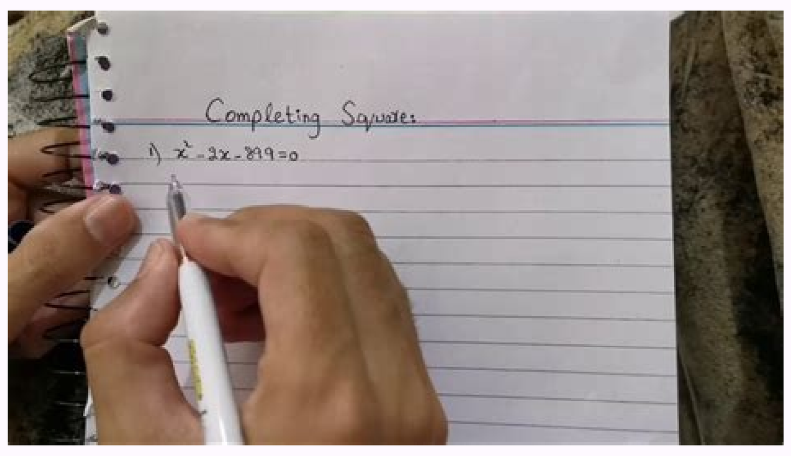

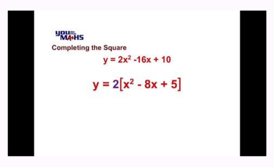

$$\begin{aligned}\int \frac{1}{x^2+6x+5}dx &= -\int \frac{1}{2^2-(x+3)^2}dx \\ &= -2\int \frac{1}{2^2-2^2\sin^2 u}\cos u\,du \\ &= -\frac{1}{2}\int \frac{1}{\cos^2 u}\cos u\,du \\ &= -\frac{1}{2}\int \sec u\,du \\ &= -\frac{1}{2}\ln|\sec u+\tan u|+C \\ &= -\frac{1}{2}\ln\left|\frac{2+(x+3)}{\sqrt{4-(x+3)^2}}\right|\end{aligned}$$

$\begingroup$ I need to complete the square on the following integral. Once this is done apparently I will be able to use on of the integration tables in the back of my book. $\int x \sqrt{x^2 + 6x +3} dx $ This is what I have come up with so far: $\int x \sqrt{(x+3)^2 -6} $ I really am at a loss. Any help with this would be appreciated. Thank you $\endgroup$ 5 "Completing the Square" is where we ... ... take a Quadratic Equation like this: and turn it into this: ax2 + bx + c = 0 a(x+d)2 + e = 0 For those of you in a hurry, I can tell you that: d = b2a and:e = c − b24a But if you have time, let me show you how to "Complete the Square" yourself. Say we have a simple expression like x2 + bx. Having x twice in the same expression can make life hard. What can we do? Well, with a little inspiration from Geometry we can convert it, like this: As you can see x2 + bx can be rearranged nearly into a square ... ... and we can complete the square with (b/2)2 In Algebra it looks like this: x2 + bx + (b/2)2 = (x+b/2)2 "Complete the Square" So, by adding (b/2)2 we can complete the square. The result of (x+b/2)2 has x only once, which is easier to use. Keeping the Balance Now ... we can't just add (b/2)2 without also subtracting it too! Otherwise the whole value changes. So let's see how to do it properly with an example: Start with: ("b" is 6 in this case) Complete the Square: Also subtract the new term Simplify it and we are done. The result: x2 + 6x + 7 = (x+3)2 − 2 And now x only appears once, and our job is done! A Shortcut Approach Here is a quick way to get an answer. You may like this method. First think about the result we want: (x+d)2 + e After expanding (x+d)2 we get: x2 + 2dx + d2 + e Now see if we can turn our example into that form to discover d and e Now we can "force" an answer: We know that 6x must end up as 2dx, so d must be 3 Next we see that 7 must become d2 + e = 9 + e, so e must be −2 And we get the same result (x+3)2 − 2 as above! Now, let us look at a useful application: solving Quadratic Equations ... Solving General Quadratic Equations by Completing the Square We can complete the square to solve a Quadratic Equation (find where it is equal to zero). But a general Quadratic Equation can have a coefficient of a in front of x2: ax2 + bx + c = 0 But that is easy to deal with ... just divide the whole equation by "a" first, then carry on: x2 + (b/a)x + c/a = 0 Steps Now we can solve a Quadratic Equation in 5 steps: Step 1 Divide all terms by a (the coefficient of x2). Step 2 Move the number term (c/a) to the right side of the equation. Step 3 Complete the square on the left side of the equation and balance this by adding the same value to the right side of the equation. We now have something that looks like (x + p)2 = q, which can be solved rather easily: Step 4 Take the square root on both sides of the equation. Step 5 Subtract the number that remains on the left side of the equation to find x. Examples OK, some examples will help! Step 1 can be skipped in this example since the coefficient of x2 is 1 Step 2 Move the number term to the right side of the equation: x2 + 4x = -1 Step 3 Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation. (b/2)2 = (4/2)2 = 22 = 4 x2 + 4x + 4 = -1 + 4 (x + 2)2 = 3 Step 4 Take the square root on both sides of the equation: x + 2 = ±√3 = ±1.73 (to 2 decimals) Step 5 Subtract 2 from both sides: x = ±1.73 – 2 = -3.73 or -0.27 And here is an interesting and useful thing. At the end of step 3 we had the equation: (x + 2)2 = 3 It gives us the vertex (turning point) of x2 + 4x + 1: (-2, -3) Step 1 Divide all terms by 5 x2 – 0.8x – 0.4 = 0 Step 2 Move the number term to the right side of the equation: x2 – 0.8x = 0.4 Step 3 Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation: (b/2)2 = (0.8/2)2 = 0.42 = 0.16 x2 – 0.8x + 0.16 = 0.4 + 0.16 (x – 0.4)2 = 0.56 Step 4 Take the square root on both sides of the equation: x – 0.4 = ±√0.56 = ±0.748 (to 3 decimals) Step 5 Subtract (-0.4) from both sides (in other words, add 0.4): x = ±0.748 + 0.4 = -0.348 or 1.148 Why "Complete the Square"? Why complete the square when we can just use the Quadratic Formula to solve a Quadratic Equation? Well, one reason is given above, where the new form not only shows us the vertex, but makes it easier to solve. There are also times when the form ax2 + bx + c may be part of a larger question and rearranging it as a(x+d)2 + e makes the solution easier, because x only appears once. For example "x" may itself be a function (like cos(z)) and rearranging it may open up a path to a better solution. Also Completing the Square is the first step in the Derivation of the Quadratic Formula Just think of it as another tool in your mathematics toolbox. 364, 1205, 365, 2331, 2332, 3213, 3896, 3211, 3212, 1206 How did I get the values of d and e from the top of the page? Start with Divide the equation by a Put c/a on other side Add (b/2a)2 to both sides "Complete the Square" Now bring everything back... ... to the left side ... to the original multiple a of x2 And you will notice that we have: Where:d = b 2a and:e = c − b2 4a Just like at the top of the page! Copyright © 2021 MathsIsFun.com By changing the square, we may rewrite any quadratic polynomial ax² + bx + c in the form \[a\left({{u^2} + {k}} \right),\;\;\text{where}\;\;u = x + \frac{b}{{2a}},\;\;k = \frac{{4ac - {b^2}}}{{4{a^2}}}.\] Completing the square helps when quadratic functions are involved in the integrand. When the integrand is a rational function with a quadratic expression in the denominator, we can use the following table integrals: \[\int {\frac{{dx}}{{\sqrt {{x^2} \pm {a^2}} }}} = \ln \left| {x + \sqrt {{x^2} \pm {a^2}} } \right|,\] \[\int {\frac{{dx}}{{\sqrt {{a^2} - {x^2}} }}} = \arcsin \frac{x}{a},\] \[\int {\frac{{dx}}{{{a^2} + {x^2}}}} = \frac{1}{a}\arctan \frac{x}{a},\] \[\int {\frac{{dx}}{{{a^2} - {x^2}}}} = \frac{1}{{2a}}\ln \left| {\frac{{a + x}}{{a - x}}} \right|.\] Certain other types of integrals involving quadratic functions can be evaluated using trigonometric and hyperbolic substitutions. Solved Problems Click or tap a problem to see the solution. Find the integral \[\int {\frac{{dx}}{{{x^2} - 5x + 7}}}.\] Evaluate the integral \[\int {\frac{{dx}}{{{x^2} - x + 2}}}.\] Evaluate the integral \[\int {\frac{{dx}}{{{x^2} + 10x + 26}}}.\] Evaluate the integral \[\int {\frac{{dx}}{{5 - 4x - {x^2}}}}.\] Find the integral \[\int {\frac{{dx}}{{\sqrt {{x^2} + x - 2} }}}.\] Find the integral \[\int {\frac{{dx}}{{8 - 2x - {x^2}}}}.\] Find the integral \[\int {\frac{{dx}}{{{x^2} - 5x + 7}}}.\] Solution. We complete the square in the denominator: \[{x^2} - 5x + 7 = {x^2} - 2 \cdot \frac{5}{2}x + {\left({\frac{5}{2}} \right)^2} - {\left({\frac{5}{2}} \right)^2} + 7 = {\left({x - \frac{5}{2}} \right)^2} + 7 - \frac{{25}}{4} = {\left({x - \frac{5}{2}} \right)^2} + \frac{{28 - 25}}{4} = {\left({x - \frac{5}{2}} \right)^2} + {\left({\frac{{\sqrt 3 }}{2}} \right)^2}.\] Make the substitution \(u = x - \frac{5}{2}.\) Then \(du = dx,\) and the integral in \(u-\)terms becomes \[\int {\frac{{dx}}{{{x^2} - 5x + 7}}} = \int {\frac{{dx}}{{{{\left({x - \frac{5}{2}} \right)}^2} + {{\left({\frac{{\sqrt 3 }}{2}} \right)}^2}}}} = \int {\frac{{du}}{{{u^2} + {{\left({\frac{{\sqrt 3 }}{2}} \right)}^2}}}} .\] We can easily evaluate the last integral: \[\int {\frac{{du}}{{{u^2} + {{\left({\frac{{\sqrt 3 }}{2}} \right)}^2}}}} = \frac{1}{{\frac{{\sqrt 3 }}{2}}}\arctan \frac{u}{{\frac{{\sqrt 3 }}{2}}} + C = \frac{2}{{\sqrt 3 }}\arctan \frac{{x - \frac{5}{2}}}{{\frac{{\sqrt 3 }}{2}}} + C = \frac{2}{{\sqrt 3 }}\arctan \frac{{2x - 5}}{{\sqrt 3 }} + C.\] Evaluate the integral \[\int {\frac{{dx}}{{{x^2} - x + 2}}}.\] Solution. We complete the square in the quadratic expression: \[{x^2} - x + 2 = {x^2} - 2 \cdot x \cdot \frac{1}{2} + {\left({\frac{1}{2}} \right)^2} - {\left({\frac{1}{2}} \right)^2} + 2 = {\left({x - \frac{1}{2}} \right)^2} - \frac{1}{4} + 2 = {\left({x - \frac{1}{2}} \right)^2} + \frac{7}{4} = {\left({x - \frac{1}{2}} \right)^2} + {\left({\frac{{\sqrt 7 }}{2}} \right)^2}.\] Making the substitution \[u = x - \frac{1}{2},\;\;du = dx,\] we find the integral: \[\int {\frac{{dx}}{{{x^2} - x + 2}}} = \int {\frac{{dx}}{{{{\left({x - \frac{1}{2}} \right)}^2} + {{\left({\frac{{\sqrt 7 }}{2}} \right)}^2}}}} = \int {\frac{{du}}{{{u^2} + {{\left({\frac{{\sqrt 7 }}{2}} \right)}^2}}}} = \frac{1}{{\frac{{\sqrt 7 }}{2}}}\arctan \frac{u}{{\frac{{\sqrt 7 }}{2}}} + C = \frac{2}{{\sqrt 7 }}\arctan \frac{{x - \frac{1}{2}}}{{\frac{{\sqrt 7 }}{2}}} + C = \frac{2}{{\sqrt 7 }}\arctan \frac{{2x - 1}}{{\sqrt 7 }} + C.\] Evaluate the integral \[\int {\frac{{dx}}{{{x^2} + 10x + 26}}}.\] Solution. We complete the square in the denominator: \[{x^2} + 10x + 26 = {x^2} + 10x + 25 + 1 = {\left({x + 5} \right)^2} + 1,\] so the integral is written as \[I = \int {\frac{{dx}}{{{x^2} + 10x + 26}}} = \int {\frac{{dx}}{{{{\left({x + 5} \right)}^2} + 1}}} .\] Using the substitution \[u = x + 5,\;\;du = dx,\] we have \[I = \int {\frac{{du}}{{{u^2} + 1}}} = \arctan u + C = \arctan \left({x + 5} \right) + C.\] Evaluate the integral \[\int {\frac{{dx}}{{5 - 4x - {x^2}}}}.\] Solution. First we complete the square in the denominator: \[5 - 4x - {x^2} = 5 - \left({{x^2} + 4x} \right) = 9 - \left({{x^2} + 4x + 4} \right) = 9 - {\left({x + 2} \right)^2} = {3^2} - {\left({x + 2} \right)^2}.\] Then the integral becomes \[I = \int {\frac{{dx}}{{5 - 4x - {x^2}}}} = \int {\frac{{dx}}{{{3^2} - {{\left({x + 2} \right)}^2}}}} .\] Using \[u = x + 2,\;\;du = dx,\] we obtain \[I = \int {\frac{{du}}{{{3^2} - {u^2}}}} = \frac{1}{{2 \cdot 3}}\ln \left| {\frac{{3 + u}}{{3 - u}}} \right| + C = \frac{1}{6}\ln \left| {\frac{{3 + x + 2}}{{3 - \left({x + 2} \right)}}} \right| + C = \frac{1}{6}\ln \left| {\frac{{5 + x}}{{1 - x}}} \right| + C.\] Find the integral \[\int {\frac{{dx}}{{\sqrt {{x^2} + x - 2} }}}.\] Solution. Complete the square in the denominator: \[{x^2} + x - 2 = \left({{x^2} + x + \frac{1}{4}} \right) - \frac{1}{4} - 2 = {\left({x + \frac{1}{2}} \right)^2} - \frac{9}{4} = {\left({x + \frac{1}{2}} \right)^2} - {\left({\frac{3}{2}} \right)^2}.\] Making the substitution \[u = x + \frac{1}{2},\;\;du = dx,\] we obtain \[\int {\frac{{dx}}{{\sqrt {{x^2} + x - 2} }}} = \int {\frac{{dx}}{{\sqrt {{{\left({x + \frac{1}{2}} \right)}^2} - {{\left({\frac{3}{2}} \right)}^2}} }}} = \int {\frac{{du}}{{\sqrt {{u^2} - {{\left({\frac{3}{2}} \right)}^2}} }}} = \ln \left| {u + \sqrt {{u^2} - {{\left({\frac{3}{2}} \right)}^2}} } \right| + C = \ln \left| {x + \frac{1}{2} + \sqrt {{x^2} + x - 2} } \right| + C.\] Find the integral \[\int {\frac{{dx}}{{8 - 2x - {x^2}}}}.\] Solution. Completing the square in the denominator yields: \[8 - 2x - {x^2} = 8 - \left({{x^2} + 2x} \right) = 9 - \left({{x^2} + 2x + 1} \right) = {3^2} - {\left({x + 1} \right)^2}.\] Hence, the integral can written in the form \[I = \int {\frac{{dx}}{{8 - 2x - {x^2}}}} = \int {\frac{{dx}}{{{3^2} - {{\left({x + 1} \right)}^2}}}} .\] Making the substitution \[u = x + 1,\;\;du = dx,\] we have \[I = \int {\frac{{dx}}{{{3^2} - {{\left({x + 1} \right)}^2}}}} = \int {\frac{{du}}{{{3^2} - {u^2}}}} = \frac{1}{{2 \cdot 3}}\ln \left| {\frac{{3 + u}}{{3 - u}}} \right| + C = \frac{1}{6}\ln \left| {\frac{{3 + x + 1}}{{3 - \left({x + 1} \right)}}} \right| + C = \frac{1}{6}\ln \left| {\frac{{4 + x}}{{2 - x}}} \right| + C.\] See more problems on Page 2. Page 2 Click or tap a problem to see the solution. Find the integral \[\int {\frac{{dx}}{{\sqrt {1 - 2x - {x^2}} }}}.\] Evaluate the integral \[\int {\frac{{x + 1}}{{{x^2} + x + 1}} dx}.\] Evaluate the integral \[\int {\frac{xdx}{{{x^2} + 2x + 10}}}.\] Compute the integral \[\int {\frac{{xdx}}{{\sqrt {5 + x - {x^2}} }}}.\] Compute the integral \[\int {\frac{{x + 1}}{{\sqrt {{x^2} + 4x + 8} }} dx}.\] Find the integral \[\int {\frac{{dx}}{{\sqrt {1 - 2x - {x^2}} }}}.\] Solution. Completing the square in the denominator, we get \[1 - 2x - {x^2} = 1 - \left({{x^2} + 2x} \right) = 2 - \left({{x^2} + 2x + 1} \right) = 2 - {\left({x + 1} \right)^2} = {\left({\sqrt 2 } \right)^2} - {\left({x + 1} \right)^2}.\] Make the substitution \[u = x + 1,\;\;du = dx.\] Hence \[\int {\frac{{dx}}{{\sqrt {1 - 2x - {x^2}} }}} = \int {\frac{{dx}}{{\sqrt {{{\left({\sqrt 2 } \right)}^2} - {{\left({x + 1} \right)}^2}} }}} = \int {\frac{{du}}{{\sqrt {{{\left({\sqrt 2 } \right)}^2} - {u^2}} }}} = \arcsin \frac{u}{{\sqrt 2 }} + C = \arcsin \frac{{x + 1}}{{\sqrt 2 }} + C.\] Evaluate the integral \[\int {\frac{{x + 1}}{{{x^2} + x + 1}} dx}.\] Solution. The quadratic function in the denominator does not have real roots, so we can't factor it. Therefore, we complete the square: \[{x^2} + x + 1 = {x^2} + x + \frac{1}{4} + \frac{3}{4} = {\left({x + \frac{1}{2}} \right)^2} + {\left({\frac{{\sqrt 3 }}{2}} \right)^2}.\] Express the numerator in terms of \({x + \frac{1}{2}}:\) \[x + 1 = \left({x + \frac{1}{2}} \right) + \frac{1}{2}.\] Using the table integrals, we get \[\int {\frac{{x + 1}}{{{x^2} + x + 1}}dx} = \int {\frac{{\left({x + \frac{1}{2}} \right) + \frac{1}{2}}}{{{x^2} + x + 1}}dx} = \int {\frac{{x + \frac{1}{2}}}{{{x^2} + x + 1}}dx} + \frac{1}{2}\int {\frac{{dx}}{{{x^2} + x + 1}}} = \frac{1}{2}\int {\frac{{\left({2x + 1} \right)dx}}{{{x^2} + x + 1}}} + \frac{1}{2}\int {\frac{{dx}}{{{{\left({x + \frac{1}{2}} \right)}^2} + {{\left({\frac{{\sqrt 3 }}{2}} \right)}^2}}}} = \frac{1}{2}\int {\frac{{d\left({{x^2} + x + 1} \right)}}{{{x^2} + x + 1}}} + \frac{1}{2}\int {\frac{{d\left({x + \frac{1}{2}} \right)}}{{{{\left({x + \frac{1}{2}} \right)}^2} + {{\left({\frac{{\sqrt 3 }}{2}} \right)}^2}}}} = \frac{1}{2}\ln \left| {{x^2} + x + 1} \right| + \frac{1}{2} \cdot \frac{1}{{\frac{{\sqrt 3 }}{2}}}\arctan \frac{{x + \frac{1}{2}}}{{\frac{{\sqrt 3 }}{2}}} + C = \frac{1}{2}\ln \left({{x^2} + x + 1} \right) + \frac{1}{{\sqrt 3 }}\arctan \frac{{2x + 1}}{{\sqrt 3 }} + C.\] Evaluate the integral \[\int {\frac{xdx}{{{x^2} + 2x + 10}}}.\] Solution. We complete the square in the denominator: \[{x^2} + 2x + 10 = {x^2} + 2x + 1 + 9 = {\left({x + 1} \right)^2} + {3^2}.\] Write the numerator in terms of \({x + 1}:\) \[x = x + 1 - 1.\] Hence, we can split the initial integral into two simpler ones: \[I = \int {\frac{xdx}{{{x^2} + 2x + 10}}} = \int {\frac{{x + 1 - 1}}{{{x^2} + 2x + 10}}dx} = \int {\frac{{\left({x + 1} \right)dx}}{{{x^2} + 2x + 10}}} - \int {\frac{{dx}}{{{x^2} + 2x + 10}}} = \frac{1}{2}\int {\frac{{\left({2x + 2} \right)dx}}{{{x^2} + 2x + 10}}} - \int {\frac{{dx}}{{{{\left({x + 1} \right)}^2} + {3^2}}}} = {I_1} - {I_2}.\] We calculate both integrals separately. In the first integral, let \(u = {x^2} + 2x + 10.\) Then \(du = \left({2x + 2} \right)dx,\) so that \[{I_1} = \frac{1}{2}\int {\frac{{\left({2x + 2} \right)dx}}{{{x^2} + 2x + 10}}} = \frac{1}{2}\int {\frac{{du}}{u}} = \frac{1}{2}\ln \left| u \right| = \frac{1}{2}\ln \left| {{x^2} + 2x + 10} \right| = \frac{1}{2}\ln \left({{x^2} + 2x + 10} \right).\] Using integration formulas from a table of integrals, we can easily evaluate the second integral: \[{I_2} = \int {\frac{{dx}}{{{{\left({x + 1} \right)}^2} + {3^2}}}} = \frac{1}{3}\arctan \frac{{x + 1}}{3}.\] Then \[I = {I_1} - {I_2} = \frac{1}{2}\ln \left({{x^2} + 2x + 10} \right) - \frac{1}{3}\arctan \frac{{x + 1}}{3} + C.\] Compute the integral \[\int {\frac{{xdx}}{{\sqrt {5 + x - {x^2}} }}}.\] Solution. Given that \[\left({5 + x - {x^2}} \right)^\prime = 1 - 2x,\] we split the numerator and write the initial integral as the sum of two integrals: \[I = \int {\frac{{xdx}}{{\sqrt {5 + x - {x^2}} }}} = - \frac{1}{2}\int {\frac{{\left({ - 2x} \right)}}{{\sqrt {5 + x - {x^2}} }}dx} = - \frac{1}{2}\int {\frac{{\left({ - 2x + 1 - 1} \right)}}{{\sqrt {5 + x - {x^2}} }}dx} = - \frac{1}{2}\int {\frac{{\left({1 - 2x} \right)}}{{\sqrt {5 + x - {x^2}} }}dx} + \frac{1}{2}\int {\frac{{dx}}{{\sqrt {5 + x - {x^2}} }}} = {I_1} + {I_2}.\] To find the first integral \({I_1},\) we use the substitution \[u = 5 + x - {x^2},\;\;du = \left({1 - 2x} \right)dx.\] Then \[{I_1} = - \frac{1}{2}\int {\frac{{\left({1 - 2x} \right)}}{{\sqrt {5 + x - {x^2}} }}dx} = - \frac{1}{2}\int {\frac{{du}}{{\sqrt u }}} = - \sqrt u = - \sqrt {5 + x - {x^2}} .\] To evaluate the second integral \({I_2},\) we complete the square in the denominator: \[5 + x - {x^2} = 5 - \left({{x^2} - x} \right) = 5 + \frac{1}{4} - \left({{x^2} - x + \frac{1}{4}} \right) = \frac{{21}}{4} - {\left({x - \frac{1}{2}} \right)^2} = {\left({\frac{{\sqrt {21} }}{2}} \right)^2} - {\left({x - \frac{1}{2}} \right)^2}.\] Now we can express the integral in terms of the inverse sine function: \[{I_2} = \frac{1}{2}\int {\frac{{dx}}{{\sqrt {5 + x - {x^2}} }}} = \frac{1}{2}\int {\frac{{dx}}{{\sqrt {{{\left({\frac{{\sqrt {21} }}{2}} \right)}^2} - {{\left({x - \frac{1}{2}} \right)}^2}} }}} = \frac{1}{2}\arcsin \frac{{x - \frac{1}{2}}}{{\frac{{\sqrt {21} }}{2}}} = \frac{1}{2}\arcsin \frac{{2x - 1}}{{\sqrt {21} }}.\] The final answer is given by \[I = - \sqrt {5 + x - {x^2}} + \frac{1}{2}\arcsin \frac{{2x - 1}}{{\sqrt {21} }} + C.\] Compute the integral \[\int {\frac{{x + 1}}{{\sqrt {{x^2} + 4x + 8} }} dx}.\] Solution. We split the numerator and write the initial integral as the sum of two integrals. Notice that \[\left({{x^2} + 4x + 8} \right)^\prime = 2x + 4.\] Then \[\int {\frac{{x + 1}}{{\sqrt {{x^2} + 4x + 8} }}dx} = \frac{1}{2}\int {\frac{{2x + 2}}{{\sqrt {{x^2} + 4x + 8} }}dx} = \frac{1}{2}\int {\frac{{2x + 4 - 2}}{{\sqrt {{x^2} + 4x + 8} }}dx} = \frac{1}{2}\int {\frac{{2x + 4}}{{\sqrt {{x^2} + 4x + 8} }}dx} - \int {\frac{{dx}}{{\sqrt {{x^2} + 4x + 8} }}} = {I_1} - {I_2}.\] The first integral \({I_1}\) is solved using the substitution \[u = {x^2} + 4x + 8,\;\;du = \left({2x + 4} \right)dx.\] Hence \[{I_1} = \frac{1}{2}\int {\frac{{2x + 4}}{{\sqrt {{x^2} + 4x + 8} }}dx} = \frac{1}{2}\int {\frac{{du}}{{\sqrt u }}} = \sqrt u = \sqrt {{x^2} + 4x + 8} .\] To find the second integral \({I_2},\) we complete the square in the denominator: \[{x^2} + 4x + 8 = \left({{x^2} + 4x + 4} \right) + 4 = {\left({x + 2} \right)^2} + {2^2}.\] Making the change \(t = x + 2,\) \(dt = dx,\) we obtain \[{I_2} = \int {\frac{{dx}}{{\sqrt {{x^2} + 4x + 8} }}} = \int {\frac{{dx}}{{\sqrt {{{\left({x + 2} \right)}^2} + {2^2}} }}} = \int {\frac{{dt}}{{\sqrt {{t^2} + {2^2}} }}} = \ln \left| {t + \sqrt {{t^2} + {2^2}} } \right| = \ln \left| {x + 2 + \sqrt {{x^2} + 4x + 8} } \right|.\] So, the initial integral is given by \[I = \sqrt {{x^2} + 4x + 8} - \ln \left| {x + 2 + \sqrt {{x^2} + 4x + 8} } \right| + C.\]

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